Descriptive complexity in number theory and dynamics

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Informally, a real number is normal in base b if in its b-ary expansion, all digits and blocks of digits occur as often as one would expect them to, uniformly at random. We will denote the set of numbers normal in base b by $\mathcal{N}(b)$. Kechris asked several questions involving descriptive complexity of sets of normal numbers. The first of these was resolved in 1994 when Ki and Linton proved that $\mathcal{N}(b)$ is $\mathbf{\Pi}^0_3$-complete. Further questions were resolved by Becher, Heiber, and Slaman who showed that $\bigcap_{b=2}^{\infty} \mathcal{N}(b)$ is $\mathbf{\Pi}^0_3$-complete and that $\bigcup_{b=2}^{\infty} \mathcal{N}(b)$ is $\mathbf{\Sigma}^0_4$-complete. Many of the techniques used in these proofs can be used elsewhere. We will discuss recent results where similar techniques were applied to solve a problem of Sharkovsky and Sivak and a question of Kolyada, Misiurewicz, and Snoha. Furthermore, we will discuss a recent result where the set of numbers that are continued fraction normal, but not normal in any base b, was shown to be complete at the expected level of $D_2(\mathbf{\Pi}^0_3)$. An immediate corollary is that this set is uncountable, a result (due to Vandehey) only known previously assuming the generalized Riemann hypothesis.

References

[1] Jackson, S., Mance, B., and Vandehey, J. (2021). On the Borel complexity of continued fraction normal, absolutely abnormal numbers. arXiv preprint arXiv:2111.11522. https://arxiv.org/abs/2111.11522

[2] Airey, D., Jackson, S., Kwietniak, D., and Mance, B. (2020). Borel complexity of sets of normal numbers via generic points in subshifts with specification. *Transactions of the American Mathematical Society*, 373(7), 4561-4584.

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